

Table 1: The standard normal ( $Z$ ) distribution

First two digits of a.bc	Second digit after decimal(c)									
	0	1	2	3	4	5	6	7	8	9
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
4.0	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00002	0.00002	0.00002	0.00002

Table 3: Student's  $t$ -distribution

<b>df</b>	<b><math>\alpha(2)</math></b>	<b>0.2</b>	<b>0.1</b>	<b>0.05</b>	<b>0.02</b>	<b>0.01</b>	<b>0.001</b>	<b>0.0001</b>
1		3.08	6.31	12.71	31.82	63.66	636.62	6366.20
2		1.89	2.92	4.30	6.96	9.92	31.60	99.99
3		1.64	2.35	3.18	4.54	5.84	12.92	28.00
4		1.53	2.13	2.78	3.75	4.60	8.61	15.54
5		1.48	2.02	2.57	3.36	4.03	6.87	11.18
6		1.44	1.94	2.45	3.14	3.71	5.96	9.08
7		1.41	1.89	2.36	3.00	3.50	5.41	7.88
8		1.40	1.86	2.31	2.90	3.36	5.04	7.12
9		1.38	1.83	2.26	2.82	3.25	4.78	6.59
10		1.37	1.81	2.23	2.76	3.17	4.59	6.21
11		1.36	1.80	2.20	2.72	3.11	4.44	5.92
12		1.36	1.78	2.18	2.68	3.05	4.32	5.69
13		1.35	1.77	2.16	2.65	3.01	4.22	5.51
14		1.35	1.76	2.14	2.62	2.98	4.14	5.36
15		1.34	1.75	2.13	2.60	2.95	4.07	5.24
16		1.34	1.75	2.12	2.58	2.92	4.01	5.13
17		1.33	1.74	2.11	2.57	2.90	3.97	5.04
18		1.33	1.73	2.10	2.55	2.88	3.92	4.97
19		1.33	1.73	2.09	2.54	2.86	3.88	4.90
20		1.33	1.72	2.09	2.53	2.85	3.85	4.84

Table 1: The  $\chi^2$  distribution

<i>df</i>	$\alpha$									
	<b>0.999</b>	<b>0.995</b>	<b>0.99</b>	<b>0.975</b>	<b>0.95</b>	<b>0.05</b>	<b>0.025</b>	<b>0.01</b>	<b>0.005</b>	<b>0.001</b>
1	0.0000016	0.000039	0.00016	0.00098	0.00393	3.84	5.02	6.63	7.88	10.83
2	0.002	0.01	0.02	0.05	0.10	5.99	7.38	9.21	10.60	13.82
3	0.02	0.07	0.11	0.22	0.35	7.81	9.35	11.34	12.84	16.27
4	0.09	0.21	0.30	0.48	0.71	9.49	11.14	13.28	14.86	18.47
5	0.21	0.41	0.55	0.83	1.15	11.07	12.83	15.09	16.75	20.52
6	0.38	0.68	0.87	1.24	1.64	12.59	14.45	16.81	18.55	22.46
7	0.60	0.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28	24.32
8	0.86	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.95	26.12
9	1.15	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59	27.88
10	1.48	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19	29.59
11	1.83	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76	31.26
12	2.21	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30	32.91
13	2.62	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82	34.53
14	3.04	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32	36.12
15	3.48	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80	37.70
16	3.94	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27	39.25
17	4.42	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72	40.79
18	4.90	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16	42.31
19	5.41	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58	43.82
20	5.92	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00	45.31

Table 1: Mann-Whitney  $U$ -distribution critical values for  $\alpha(2) = 0.05$

$n_2$	$n_1$												
	3	4	5	6	7	8	9	10	11	12	13	14	15
3	–	–	15	17	20	22	25	27	30	32	35	37	40
4	–	16	19	22	25	28	32	35	38	41	44	47	50
5	15	19	23	27	30	34	38	42	46	49	53	57	61
6	17	22	27	31	36	40	44	49	53	58	62	67	71
7	20	25	30	36	41	46	51	56	61	66	71	76	81
8	22	28	34	40	46	51	57	63	69	74	80	86	91
9	25	32	38	44	51	57	64	70	76	82	89	95	101
10	27	35	42	49	56	63	70	77	84	91	97	104	111
11	30	38	46	53	61	69	76	84	91	99	106	114	121
12	32	41	49	58	66	74	82	91	99	107	115	123	131
13	35	44	53	62	71	80	89	97	106	115	124	132	141
14	37	47	57	67	76	86	95	104	114	123	132	141	151
15	40	50	61	71	81	91	101	111	121	131	141	151	161

Table 2: Mann-Whitney  $U$ -distribution critical values for  $\alpha(2) = 0.01$

$n_2$	$n_1$												
	3	4	5	6	7	8	9	10	11	12	13	14	15
3	–	–	–	–	–	–	27	30	33	35	38	41	43
4	–	–	–	24	28	31	35	38	42	45	49	52	55
5	–	–	25	29	34	38	42	46	50	54	58	63	67
6	–	24	29	34	39	44	49	54	59	63	68	73	78
7	–	28	34	39	45	50	56	61	67	72	78	83	89
8	–	31	38	44	50	57	63	69	75	81	87	94	100
9	27	35	42	49	56	63	70	77	83	90	97	104	111
10	30	38	46	54	61	69	77	84	92	99	106	114	121
11	33	42	50	59	67	75	83	92	100	108	116	124	132
12	35	45	54	63	72	81	90	99	108	117	125	134	143
13	38	49	58	68	78	87	97	106	116	125	135	144	153
14	41	52	63	73	83	94	104	114	124	134	144	154	164
15	43	55	67	78	89	100	111	121	132	143	153	164	174

<b>Population Parameters</b>												
Pop. mean: $\mu$	Pop. variance: $\sigma^2$	Pop. Std. Dev: $\sigma$	Pop. Proportion: $p$									
<b>Descriptive Sample Statistics</b>												
Sample Size: $n$	Sample variance: $s^2 = \frac{\sum(Y_i - \bar{Y})^2}{n - 1}$	Sample Standard Dev: $s = \sqrt{\frac{\sum(Y_i - \bar{Y})^2}{n - 1}}$	Coefficient of Variation $CV = \frac{s}{\bar{Y}} \times 100\%$									
Sample mean: $\bar{Y} = \frac{\sum_{i=1}^n Y_i}{n}$												
Sample median:  ( $n = \text{odd \#}$ ): $Y_{[(n+1)/2]}$  ( $n = \text{even \#}$ ): $[Y_{(n/2)} + Y_{(n/2+1)}]/2$	Estimate of proportion:  $\hat{p} = \frac{\# \text{ in category } (X \text{ succ.})}{n}$	Standard error of sample mean:  $\sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}}$	Estimate of standard error of sample mean:  $SE_{\bar{Y}} = \frac{s}{\sqrt{n}}$									
<b>Probability Rules</b>												
Pr[A or B] = Pr[A] + Pr[B] - Pr[A and B], Pr[A and B] = 0 when A and B are mutually exclusive.												
Pr[A and B] = Pr[A]Pr[B A], Pr[A and B] = Pr[A] X Pr[B] when A and B are independent.												
Law of Total Probability $Pr[A] = \sum_{\text{All values of B}} Pr[B]Pr[A B]$		Bayes' Theorem $Pr[A B] = \frac{Pr[B A]Pr[A]}{Pr[B]}$										
<b>Proportions</b>												
Binomial Formula:  $Pr[X \text{ successes}] = \binom{n}{X} p^X (1 - p)^{n-X}$	Factorial Formula:  $\binom{n}{X} = \frac{n!}{X!(n - X)!}$	Estimate - Standard error of $\hat{p}$ :  $SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$										
Agresti-Coull 95% C.I. for proportion $p$ :  $p' - 1.96 \sqrt{\frac{p'(1 - p')}{n + 4}} < p < p' + 1.96 \sqrt{\frac{p'(1 - p')}{n + 4}}$ , where $p' = \frac{X + 2}{n + 4}$												
<b>Categorical Data: Goodness-of-fit and contingency tables</b>												
$\chi^2 = \sum_i \frac{(\text{Observed}_i - \text{Expected}_i)^2}{\text{Expected}_i}$ , where $df = (\# \text{ of categories}) - 1 - (\# \text{ estimated parameters})$												
Odds of success: $O = \frac{p}{1 - p}$	Estimate of odds: $\hat{O} = \frac{\hat{p}}{1 - \hat{p}}$	Odds ratio: $OR = \frac{O_1}{O_2}$	<table border="1"> <thead> <tr> <th></th> <th>Treat</th> <th>Control</th> </tr> </thead> <tbody> <tr> <th>Succ.</th> <td>a</td> <td>b</td> </tr> <tr> <th>Fail.</th> <td>c</td> <td>d</td> </tr> </tbody> </table> $\widehat{OR} = \frac{a/c}{b/d} = \frac{a d}{b c}$		Treat	Control	Succ.	a	b	Fail.	c	d
	Treat	Control										
Succ.	a	b										
Fail.	c	d										
$SE[\ln(\widehat{OR})]$  $= \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$	100(1- $\alpha$ )% C.I. for the log-odds ratio:  $\ln(\widehat{OR}) - Z \times SE[\ln(\widehat{OR})] < \ln(OR) < \ln(\widehat{OR}) + Z \times SE[\ln(\widehat{OR})]$ where Z is 1.96 for 95% C.I., 2.58 for 99% C.I.											
contingency table $df = (r - 1)(c - 1)$ , $r = \text{rows}$ , and $c = \text{columns}$												
$Expected[\text{row } i, \text{column } j] = \frac{(\text{Row total } i)}{\text{Grand total}} \times \text{Column Total } j$												

**Normal distribution:**

$$f(Y | \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(Y-\mu)^2}{2\sigma^2}} \quad \sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}} \quad Z = \frac{\bar{Y}-\mu}{\sigma_{\bar{Y}}}$$

When appropriate, probabilities for a binomial distribution for  $n$  trials and a probability of success  $p$  can be approximated by a Normal( $\mu = np, \sigma = \sqrt{np(1-p)}$ ) distribution.

**Student's  $t$  distributions for one mean:**

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}} \quad t = \frac{\bar{Y}-\mu}{SE_{\bar{Y}}} \quad df = n - 1$$

100(1 -  $\alpha$ )% C. I. of the mean:  $\bar{Y} - t_{\alpha(2),df}SE_{\bar{Y}} < \mu < \bar{Y} + t_{\alpha(2),df}SE_{\bar{Y}}$

**Two sample  $t$  test for difference of means:**

$$SE_{\bar{Y}_1-\bar{Y}_2} = \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} \quad s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} \quad t = \frac{\bar{Y}_1-\bar{Y}_2-(\mu_1-\mu_2)}{SE_{\bar{Y}_1-\bar{Y}_2}} \quad df = n_1 + n_2 - 2$$

100(1 -  $\alpha$ )% C. I. of the difference in means:

$$\bar{Y}_1 - \bar{Y}_2 - t_{\alpha(2),df}SE_{\bar{Y}_1-\bar{Y}_2} < \mu_1 - \mu_2 < \bar{Y}_1 - \bar{Y}_2 + t_{\alpha(2),df}SE_{\bar{Y}_1-\bar{Y}_2}$$

**log transformation:**

$$Y'_i = \ln Y_i \quad \text{back transform: } Y_i = e^{Y'_i} \quad \ln A - \ln B = \ln \left( \frac{A}{B} \right)$$

**Mann-Whitney  $U$ :**

$R_1$  is the sum of ranks for category 1.

$$U_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1 \quad U_2 = n_1n_2 - U_1 \quad U = \max(U_1, U_2)$$