

Question 19 of chapter 5 explains the terminology that we use when we talk about decks of cards. Imagine that your opponent will draw a single card randomly from a deck.

Half point for each problem. Broken up as: .1 point for each simple probability extracted from the question (0.2). 0.1 point for recognizing the OR connection. 0.1 point for mutual-exclusivity/rule choice; 0.1 for the correct final answer.

**Question #1:** If your opponent draws a diamond face card or any spade, then she will win. What is the probability that she will win?

$c$  denotes the card drawn

$$\Pr(c \text{ is a diamond face card OR } c \text{ is a spade}) = ?$$

$c$  cannot be both a diamond and spade  $\rightarrow$  the events are mutually exclusive

$$\Pr(c \text{ is a diamond face card OR } c \text{ is a spade}) = \Pr(c \text{ is a diamond face card}) + \Pr(c \text{ is a spade})$$

$$\Pr(c \text{ is a diamond face card}) = \frac{3}{52}$$

$$\Pr(c \text{ is a spade}) = \frac{13}{52}$$

$$\Pr(c \text{ is a diamond face card OR } c \text{ is a spade}) = \frac{16}{52} = \frac{4}{13}$$

**Question #2:** In another game, if your opponent draws *any* face card or any spade, then she will win. What is the probability that she will win?

$c$  denotes the card drawn

$$\Pr(c \text{ is a face card OR } c \text{ is a spade}) = ?$$

$c$  can be both a face card and spade  $\rightarrow$  the events are not mutually exclusive

$$\Pr(c \text{ is a face card OR } c \text{ is a spade}) = \Pr(c \text{ is a face card}) + \Pr(c \text{ is a spade}) - \Pr(c \text{ is spade face card})$$

$$\Pr(c \text{ is a face card}) = \frac{12}{52}$$

$$\Pr(c \text{ is a spade}) = \frac{13}{52}$$

$$\Pr(c \text{ is a spade face card}) = \frac{3}{52}$$

$$\Pr(c \text{ is a face card OR } c \text{ is a spade}) = \frac{12 + 13 - 3}{52} = \frac{11}{26}$$

**Question #3:** A bag has 2 black marbles and 7 white marbles. You reach into the bag and draw a marble. Then you return the marble to the bag, shake the bag (to randomize it), and draw another marble. You do this until you have drawn a total of four marbles.

- a. What is the probability that the first draw is a white marble (Write this as an equation using probability notation).
- b. What is the probability that the first draw is a black marble. For each draw the sample space is  $\{B, W\}$ . The information about the number of marbles of each color in the bag is telling us about the relative frequency of the color. This is the probability of drawing a marble of that color on the first draw.

$$\Pr(W) = \frac{7}{9}$$

$$\Pr(B) = \frac{2}{9}$$

- c. What is the chance that all four draws were white?
- d. What is the chance that all four draws were black?

The sample space for draws includes 16 outcomes (you would not have to list these in a test)

$$S = \{BBBB, BBBW, BBWB, BBWW, BWBB, BWBW, BWWB, BWWW, WBBB \dots WWWW\}$$

(note that these sum to one. Having identified the sample space for a single draw lets us double check our numbers). The fact that we are replacing the marbles, mean that we return the population to its original state. Each draw will be independent

$A$  is the event that all 4 draws are white.

$$\Pr(A) = \Pr(d_1 = W \text{ and } d_2 = W \text{ and } d_3 = W \text{ and } d_4 = W)$$

We are not picky:

$$\Pr(A) = \Pr(W \text{ and } W \text{ and } W \text{ and } W)$$

is fine when you are in a hurry (but much less clear).

We use the simplest form of multiplication rule because the events are independent

$$\begin{aligned} \Pr(d_1 = W \text{ and } d_2 = W \text{ and } d_3 = W \text{ and } d_4 = W) &= \Pr(d_1 = W) \Pr(d_2 = W) \Pr(d_3 = W) \Pr(d_4 = W) \\ &= \left(\frac{7}{9}\right)^4 \approx 0.366 \\ \Pr(d_1 = B \text{ and } d_2 = B \text{ and } d_3 = B \text{ and } d_4 = B) &= \Pr(d_1 = B) \Pr(d_2 = B) \Pr(d_3 = B) \Pr(d_4 = B) \\ &= \left(\frac{2}{9}\right)^4 \approx 0.00244 \end{aligned}$$

**Question #4:** In population genetics we often calculate the probability of a zygote's genotype, by treating alleles contributed by each parent as random draws from the same pool of alleles (this is the random mating assumption). A particular locus has a common allele (T) at frequency of 90% and one other allele (t). Assume that the organism is diploid and the gene is autosomal (in other words, a zygote's genotype consists of two copies of the gene).

a) What is the probability that a zygote will have at least one T allele ?

if  $g$  is the genotype, and  $g_p$  and  $g_m$  represent the paternal and maternal contributions to the genotype:

$$\begin{aligned}
 \Pr(g = \text{not } tt) &= 1 - \Pr(g = tt) \\
 \Pr(g = tt) &= \Pr(g_p = t \text{ AND } g_m = t) \\
 &= \Pr(g_p = t) \Pr(g_m = t) && \text{by independence} \\
 \Pr(g_p = t) = \Pr(g_m = t) &= 1 - \Pr(g_p = T) \\
 &= 1 - 0.9 = 0.1 \\
 \Pr(g = tt) &= 0.1 \times 0.1 = 0.01 \\
 \Pr(g = \text{not } tt) &= 1 - 0.01 = 0.99
 \end{aligned}$$

b) What is the probability that mother and father contribute the same allele to the zygote (in other words, what is the chance that the zygote will be homozygous at this locus)?

Above we showed that  $\Pr(g = tt) = 0.01$

$$\begin{aligned}
 \Pr(g = tt \text{ OR } g = TT) &= \Pr(g = tt) + \Pr(g = TT) \\
 \Pr(g = TT) &= \Pr(g_p = T \text{ AND } g_m = T) \\
 &= \Pr(g_p = T) \Pr(g_m = T) \\
 \Pr(g_p = T) = \Pr(g_m = T) &= 0.9 \\
 \Pr(g = TT) &= 0.9 \times 0.9 = 0.81 \\
 \Pr(g = tt \text{ OR } g = TT) &= 0.01 + 0.81 = 0.82
 \end{aligned}$$

**Question #1:** Pit vipers are very effective predators of rodents. We will assume that a pit viper waiting along a rodent trail will always kill and eat a rodent that passes by. Based on random sampling along a particular trail, a mammalogist tells us that there is a 5% chance that the trail will be used by a cotton rat (*Sigmodon hispidus*) on any night. Deer mice (*Peromyscus maniculatus*) are more frequent, and a deer mouse will travel along the trail during 25% of all nights. No other rodents are found in this location. You can assume that the rodents do not alter their behavior based on whether another rodent (or snake) has used the trail.

- a. If a viper waits along the trail for **one night**, what is the probability that it will get a meal (that it will eat either of these two rodent species)?

$$S_1 = \{c, \text{ not } C\}$$

meaning “cotton rat uses trail” and “cotton rat does not use the trail”. For the deer mice:

$$S_2 = \{d, \text{ not } D\}$$

$$\Pr(E_1 = c) = 0.05$$

$$\Pr(E_2 = d) = 0.25$$

$$S_3 = \{\text{meal}, \text{ not meal}\}$$

$$E_3 = \text{meal} = c \text{ or } d$$

$c$  and  $d$  are **not** mutually exclusive, so we have to use the general form of the addition rule:

$$\Pr(E_3) = \Pr(c \text{ or } d) = \Pr(c) + \Pr(d) - \Pr(c \text{ and } d)$$

The statement about not altering behavior implies statistical independence, so:

$$\begin{aligned}\Pr(c \text{ and } d) &= \Pr(c) \Pr(d) \\ &= 0.05 \times 0.25 \\ &= 0.0125\end{aligned}$$

$$\begin{aligned}\Pr(\text{meal}) = \Pr(c \text{ or } d) &= \Pr(c) + \Pr(d) - \Pr(c \text{ and } d) \\ &= 0.05 + 0.25 - 0.0125 \\ &= 0.2875\end{aligned}$$

The probability of a viper getting a meal is 0.2875.

- b. If a viper is willing to wait along the trail for up to **two nights**, what is the probability that it will leave hungry (in other words the probability that it will **not** catch a rodent on **either** night)?

$$S_4 = \{[m, m], [m, n], [n, m], [n, n]\}$$

$$E_4 = \{[n, n]\}$$

Assuming independence,

$$\Pr([n, n]) = \Pr(n) \Pr(n)$$

$$\Pr(n) = 1 - \Pr(\text{meal})$$

$$= 1 - 0.2875$$

$$= 0.7125$$

$$\Pr([n, n]) = \Pr(n) \Pr(n)$$

$$= 0.7125 \times 0.7125$$

$$= 0.5076$$

The probability of a viper getting no meals after two nights is 0.5076.

**Question #2:** Consider a plant with incomplete dominance in floral color.  $RR$  individuals have red flowers,  $rr$  individuals have white flowers, and heterozygotes have pink flowers. Imagine that, in one season, 80% of red flowers will be pollinated, 60% of pink flowers will be pollinated, and only 35% of white flowers will be pollinated.

You cross two heterozygotes to produce an F1 plant which you observe over the course of a season. What is the probability that the F1 will **not** be pollinated? Recall that in Mendelian genetics problems, we treat the allelic contributions made each parents as statistically independent events.

Pollination  $p$  is a random event in the sample space  $\{Y, N\}$  (standing for Yes and No).

Color  $c$  is a random event in the sample space  $\{R, P, W\}$  (standing for Red, Pink, and White).

Genotype  $g$  of the F1 is random event in the sample space  $\{RR, Rr, rr\}$ .

$$\begin{aligned}
 \Pr(p = N) &= ? \\
 \Pr(p = Y|c = R) &= 0.8 \\
 \Pr(p = Y|c = P) &= 0.6 \\
 \Pr(p = Y|c = W) &= 0.35 \\
 \Pr(c = R|g = RR) &= 1.0 \\
 \Pr(c = P|g = Rr) &= 1.0 \\
 \Pr(c = W|g = rr) &= 1.0 \\
 \Pr(g = RR) = \Pr(c = R) &= 0.25 \\
 \Pr(g = Rr) = \Pr(c = P) &= 0.5 \\
 \Pr(g = rr) = \Pr(c = W) &= 0.25 \\
 \Pr(p = N) &= 1 - \Pr(p = Y)
 \end{aligned}$$

the last step uses the complement rule. The law of total probability tells us that:

$$\Pr(p = Y) = \sum_{C \in \{R, P, W\}} \Pr(c = C) \Pr(p = Y|c = C)$$

where  $C \in \{R, P, W\}$  just means that we sum over all 3 color outcomes (it would be read as “all outcomes  $C$  in the set  $\{R, P, W\}$ ”). You don’t have to use this style of notation, just know how to apply the law of total probability.

$$\begin{aligned}
 \Pr(p = Y) &= 0.25 \times 0.8 + 0.5 \times 0.6 + 0.25 \times 0.35 \\
 &= 0.5875 \\
 \Pr(p = N) &= 1 - 0.5875 \\
 &= 0.4125
 \end{aligned}$$