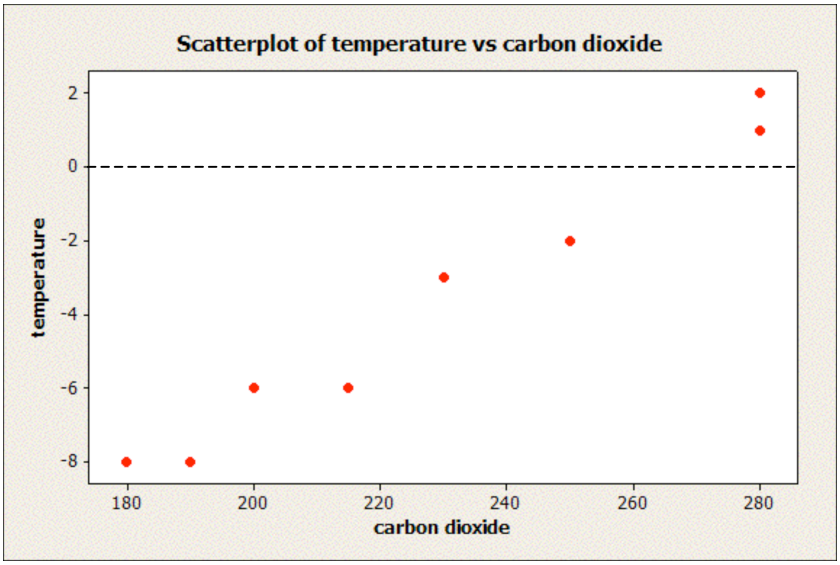
Spearman's rank correlation

Returning to the data from the Vostok ice core:



1. Calculate Spearman's rank correlation coefficient (r_S)

Carbon dioxide	Rank	Temperature	Rank					
X	R_1	Y	R_2	$R_1 - \bar{R}_1$	$R_2 - \bar{R}_2$	$(R_1 - \bar{R}_1)^2$	$(R_2 - \bar{R}_2)^2$	$(R_1 - \bar{R}_1)(R_2 - \bar{R}_2)$
280	7.5	2	8	3	3.5	9	12.25	10.5
180	1	-8	1.5	-3.5	-3	12.25	9	10.5
250	6	-2	6	1.5	1.5	2.25	2.25	2.25
200	3	-6	3.5	-1.5	-1	2.25	1	1.5
190	2	-8	1.5	-2.5	-3	6.25	9	7.5
230	5	-3	5	0.5	0.5	0.25	0.25	0.25
215	4	-6	3.5	-0.5	-1	0.25	1	0.5
280	7.5	1	7	3	2.5	9	6.25	7.5
	Mean:		Mean:			Sum:	Sum:	Sum:
	4.5		4.5			41.5	41	40.5

$$r_S = \frac{\sum (R_1 - \bar{R}_1)(R_2 - \bar{R}_2)}{\sqrt{\sum (R_1 - \bar{R}_1)^2}\sqrt{\sum (R_2 - \bar{R}_2)^2}} = \frac{40.5}{\sqrt{41.5}\sqrt{41}} = \frac{40.5}{41.249} = 0.982$$

2. Hypothesis test for the Spearman's correlation coefficient for the population (ρ_S)

Step 1: Is there a correlation between carbon dioxide and temperature change?

Step 2: H_0: There is no correlation between carbon dioxide and temperature change ($\rho_S = 0$).
H_A: There is a correlation between carbon dioxide and temperature change ($\rho_S \neq 0$).

Steps 3 and 4: Collect a random sample and calculate descriptive statistics (see above).

Step 5: Determine how compatible data are with results expected under H_0.

To determine the *P*-value for the test, compare r_S to the critical value given in Statistical Table G corresponding to the appropriate sample size, *n*.

Table G – Critical values for Spearman's rank correlation

n	$\alpha = 0.05$	$\alpha = 0.01$
5	0.900	
6	0.943	1.000
7	0.821	0.929
8	0.762	0.881
9	0.700	0.833
10	0.648	0.782
11	0.618	0.755
12	0.587	0.720
13	0.560	0.692
14	0.538	0.670
15	0.521	0.645
16	0.503	0.626
17	0.485	0.610
18	0.472	0.593
19	0.458	0.579
20	0.447	0.564
21	0.435	0.551
22	0.425	0.539
23	0.415	0.528
24	0.406	0.516
25	0.398	0.506
26	0.389	0.497
27	0.382	0.488
28	0.375	0.479
29	0.368	0.471
30	0.362	0.464
31	0.356	0.456
32	0.350	0.449
33	0.345	0.443
34	0.339	0.436
35	0.334	0.430
36	0.329	0.424
37	0.325	0.419
38	0.320	0.413
39	0.316	0.408
40	0.312	0.403
41	0.308	0.398
42	0.305	0.393
43	0.301	0.389
44	0.298	0.385
45	0.294	0.380
46	0.291	0.376
47	0.288	0.372
48	0.285	0.369
49	0.282	0.365
50	0.279	0.361
51	0.276	0.358
52	0.273	0.354
53	0.271	0.351
54	0.268	0.348
55	0.266	0.345
56	0.263	0.342
57	0.261	0.339
58	0.259	0.336
59	0.256	0.333
60	0.254	0.330
61	0.252	0.327
62	0.250	0.325
63	0.248	0.322
64	0.246	0.320
65	0.244	0.317
66	0.242	0.315
67	0.241	0.313
68	0.239	0.310
69	0.237	0.308
70	0.235	0.306
71	0.234	0.304
72	0.232	0.302
73	0.230	0.300
74	0.229	0.298
75	0.227	0.296
76	0.226	0.294
77	0.224	0.292
78	0.223	0.290
79	0.221	0.288
80	0.220	0.286
81	0.219	0.285
82	0.217	0.283
83	0.216	0.281
84	0.215	0.280
85	0.213	0.278
86	0.212	0.276
87	0.211	0.275
88	0.210	0.273
89	0.208	0.272
90	0.207	0.270
91	0.206	0.269
92	0.205	0.267
93	0.204	0.266
94	0.203	0.264
95	0.202	0.263
96	0.201	0.262
97	0.200	0.260
98	0.199	0.259
99	0.198	0.258
100	0.197	0.257

Since $r_{S(0.05,8)} = 0.762$ and $r_{S(0.01,8)} = 0.881$, and our value of $r_S = 0.982$, *P*-value < 0.01.

Step 6: Make a decision regarding H_0. *P*-value < α, we reject H_0.

Step 7: Conclusions.

With $\alpha = 0.05$, we have significant evidence (Spearman's rank correlation $r_S = 0.982$, *P*-value < 0.01) from 8 ice core samples that there is a positive correlation between carbon dioxide concentration in the atmosphere and temperature change.