

	<b>PLACEBO</b>	<b>DRUG</b>	<b>ASPIRIN</b>	
	<b>0.0</b>	<b>2.3</b>	<b>3.1</b>	
	<b>1.0</b>	<b>3.5</b>	<b>2.7</b>	
		<b>2.8</b>	<b>3.8</b>	
		<b>2.5</b>		
$\bar{Y}_i$	<b>0.5</b>	<b>2.78</b>	<b>3.2</b>	$\bar{Y} = 2.41$
$n_i$	<b>2</b>	<b>4</b>	<b>3</b>	$N = 9$
$s_i$	<b>0.707</b>	<b>0.525</b>	<b>0.557</b>	

$$H_0: \mu_1 = \mu_2 = \mu_3$$

$H_A$ : At least one of the  $\mu_i$  is different from the rest.

Statistical test: ANOVA,  $\alpha = 0.05$

**SS(error)**

$$\sum_i (n_i - 1)s_i^2 = (1)(0.707)^2 + (3)(0.525)^2 + (2)(0.557)^2 = 1.947$$

$$\mathbf{df(error)} = N - k = 9 - 3 = 6$$

$$\mathbf{MS(error)} = \text{SS(error)}/\text{df(error)} = 1.947/6 = 0.325$$

**SS(groups)**

$$\sum_i n_i (\bar{Y}_i - \bar{Y})^2 = 2(0.5 - 2.41)^2 + 4(2.78 - 2.41)^2 + 3(3.2 - 2.41)^2 = 9.716$$

$$\mathbf{df(groups)} = k - 1 = 3 - 1 = 2$$

$$\mathbf{MS(groups)} = \text{SS(groups)}/\text{df(groups)} = 9.716/2 = 4.858$$

**SS(total)**

$$\sum_i \sum_j (Y_{ij} - \bar{Y})^2 = (0.0 - 2.41)^2 + (1.0 - 2.41)^2 + \dots = 11.65$$

$$\mathbf{df(total)} = N - 1 = 9 - 1 = 8$$

$$\mathbf{SS(total)} = \text{SS(groups)} + \text{SS(error)} = 9.716 + 1.947 = 11.66$$

### ANOVA Table

<b>SOURCE</b>	<b>SS</b>	<b>df</b>	<b>MS</b>	<b><math>F_s = MS_{\text{groups}}/MS_{\text{error}}</math></b>	<b>P-value</b>
<b>Groups</b>	<b>9.716</b>	<b>2</b>	<b>4.858</b>	<b>14.95</b>	<b>&lt; 0.01</b>
<b>Error</b>	<b>1.947</b>	<b>6</b>	<b>0.325</b>		
<b>TOTAL</b>	<b>11.66</b>	<b>8</b>			

P-value, using Table D:

$$F_{0.05(1),2,6} = 5.14$$

$$F_{0.01(1),2,6} = 10.92$$

$$P\text{-value} < 0.01$$

**P-value <  $\alpha$ , we reject  $H_0$**

In a study comparing 3 pain treatments (placebo, drug, aspirin), there is sufficient evidence to reject the null hypothesis that the three treatments give equal numbers of hours free of pain ( $F_{2,6} = 14.95$ ,  $P\text{-value} < 0.01$ ). At least one of the means is different (placebo  $\bar{Y}_1 = 0.5$  hours,  $s_1 = 0.707$ ,  $n_1 = 2$ ; drug  $\bar{Y}_2 = 2.78$  hours,  $s_2 = 0.525$ ,  $n_2 = 4$ ; aspirin  $\bar{Y}_3 = 3.2$  hours,  $s_3 = 0.557$ ,  $n_3 = 3$ ).

### **Variability explained – $R^2$**

$$R^2 = \frac{SS_{\text{groups}}}{SS_{\text{total}}} = \frac{9.716}{11.66} = 0.833$$

Approximately 83% of the variability among subjects in the hours of pain relief after treatment is explained by the different treatments. The remaining approximately 17% of the variability is not explained by the treatments; it is random variability (“error”).