95% confidence interval for the difference between two sample means.

We are 95% confident that $\mu_1 - \mu_2$ is between:

$$(\bar{Y}_1 - \bar{Y}_2) - \left(SE_{\bar{Y}_1 - \bar{Y}_2}\right) t_{0.05(2),df}$$

and

$$(\bar{Y}_1 - \bar{Y}_2) + \left(SE_{\bar{Y}_1 - \bar{Y}_2}\right) t_{0.05(2),df}$$

In the sparrow example, we are 95% confident that:

$$0.64 - 0.248 * 1.98 < \mu_d - \mu_s < 0.64 + 0.248 * 1.98$$

$$0.143 < \mu_d - \mu_s < 1.126$$

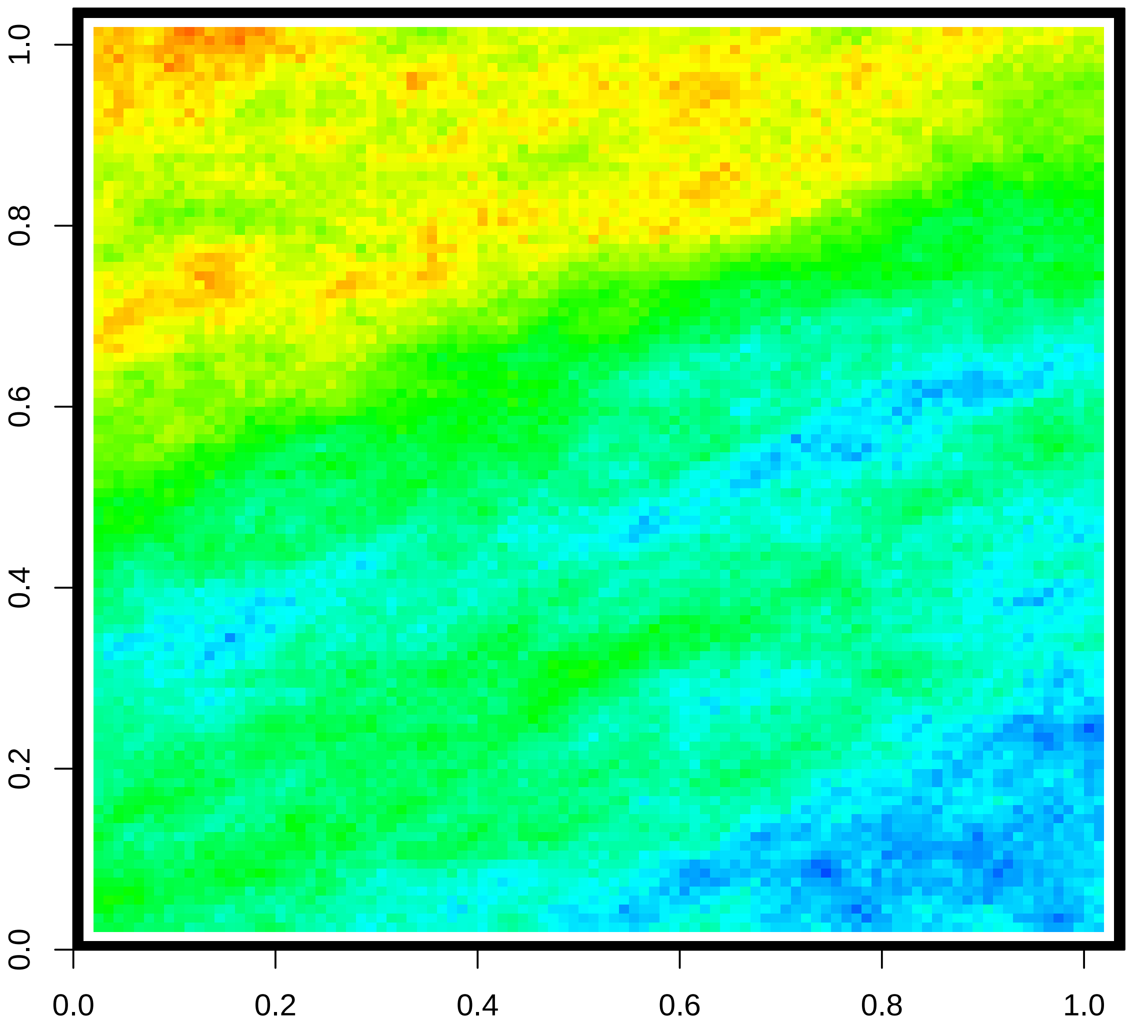
Why do we calculate the variance of the population using:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

instead of calculating a global mean and standard deviation using all of the data?

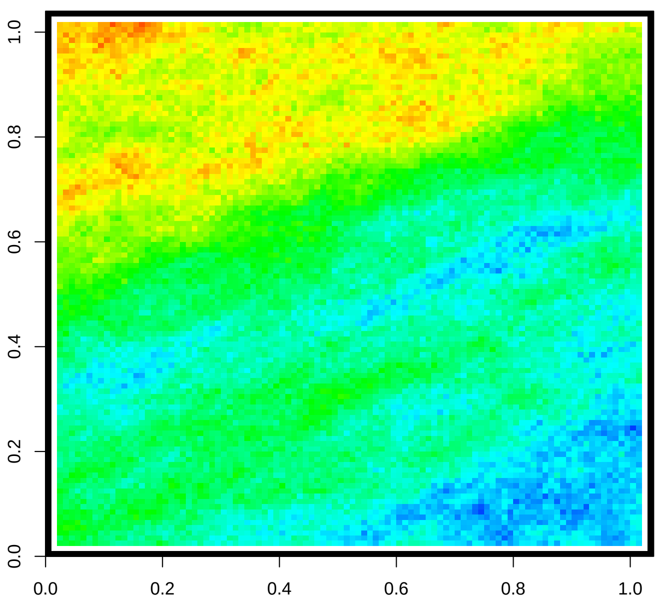
red $\approx$ 10 expected

blue $\approx$ 250 expected

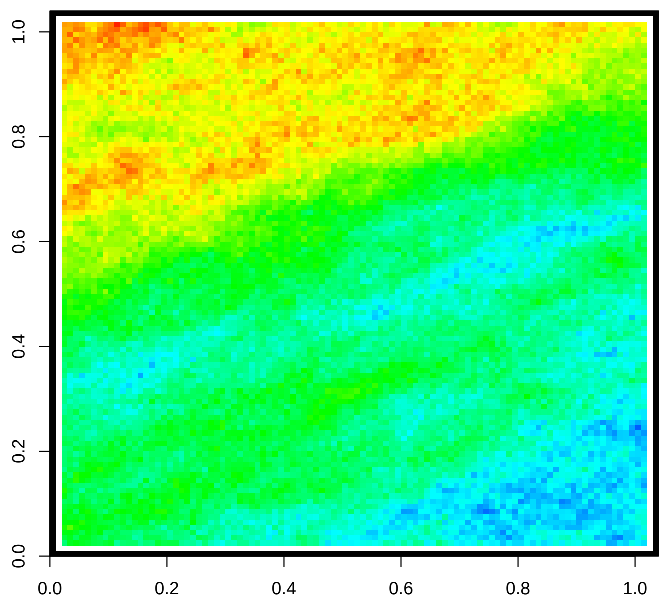


If we could treat the entire landscape, we would see a small effect:

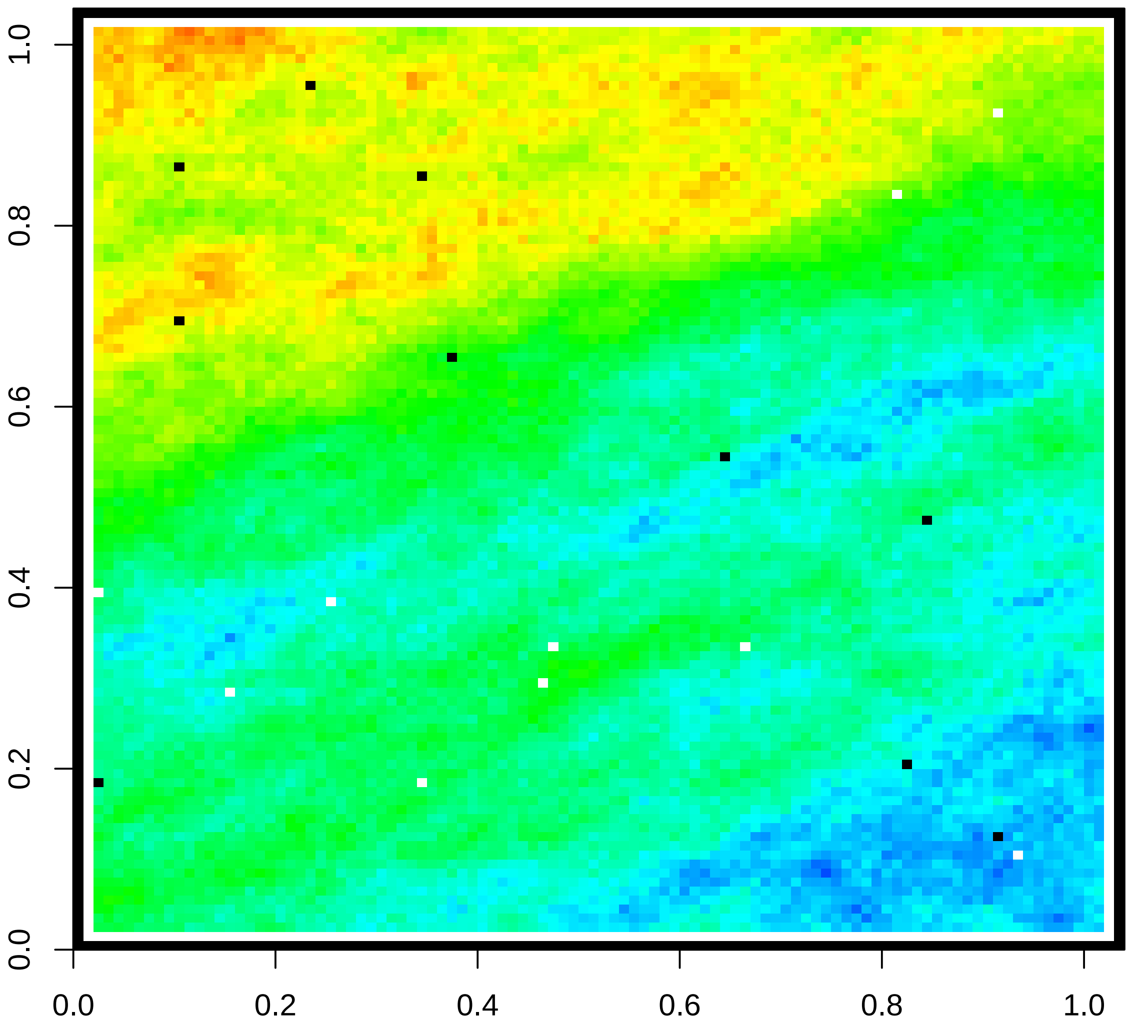
Control:



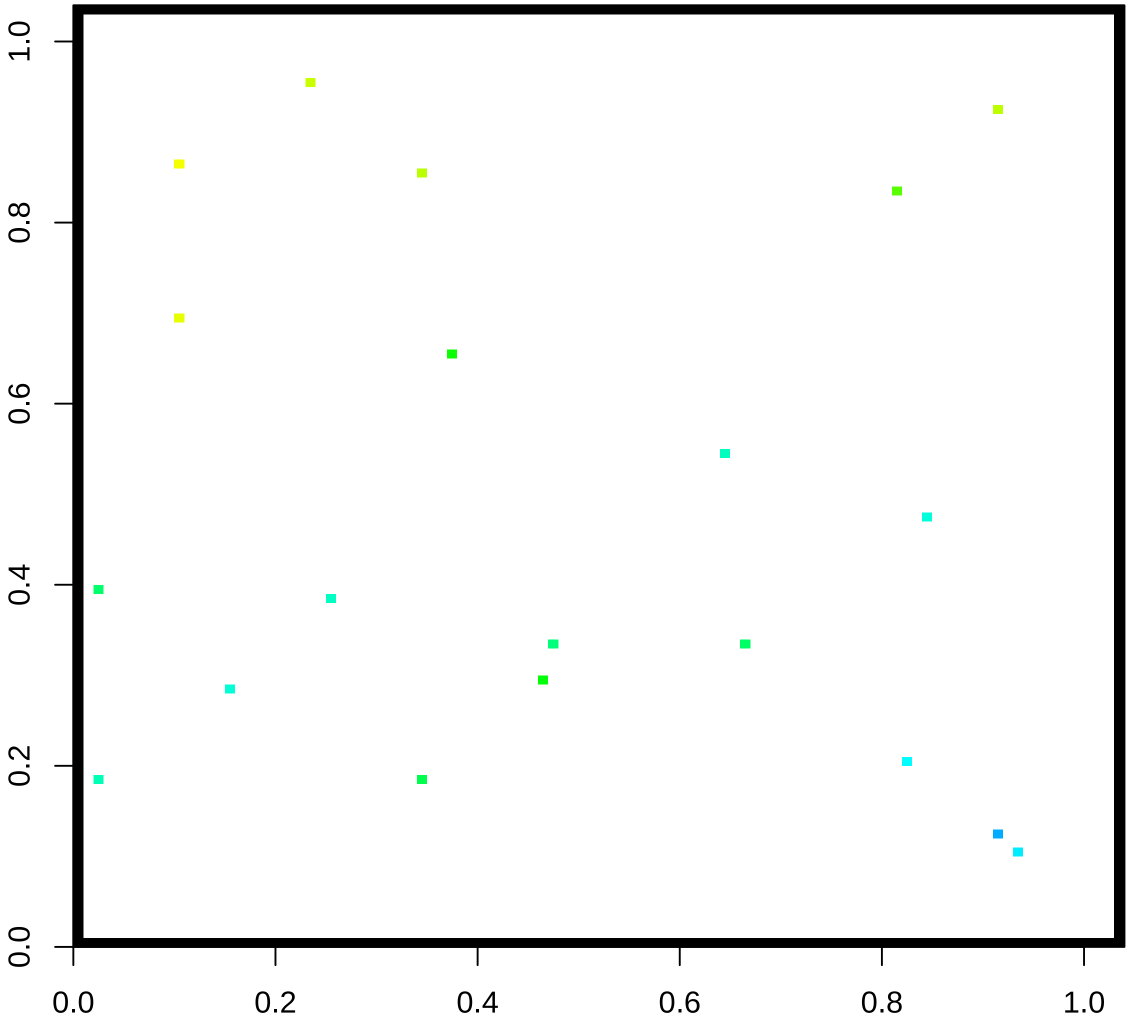
Treatment:



We could randomly select 10 control locations and 10 treatment locations:



Only the sampled locations shown:



Treatment	Control
173.9	202.0
187.3	119.9
168.0	74.1
77.1	167.1
151.2	64.7
101.5	174.8
147.9	79.0
124.3	165.5
146.0	183.6
140.8	67.5

$\bar{Y}_T = 141.80 \qquad \sigma_T^2 = 1116.4 \qquad n_T = 10$

$\bar{Y}_C = 129.82 \qquad \sigma_C^2 = 2966.5 \qquad n_C = 10$

a two-sample t-test:

$$\bar{Y}_T = 141.80 \qquad \sigma_T^2 = 1116.4 \qquad n_T = 10$$

$$\bar{Y}_C = 129.82 \qquad \sigma_C^2 = 2966.5 \qquad n_C = 10$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$s_p^2 = \frac{(9)1116.4 + (9)2966.5}{18} = 2041.5$$

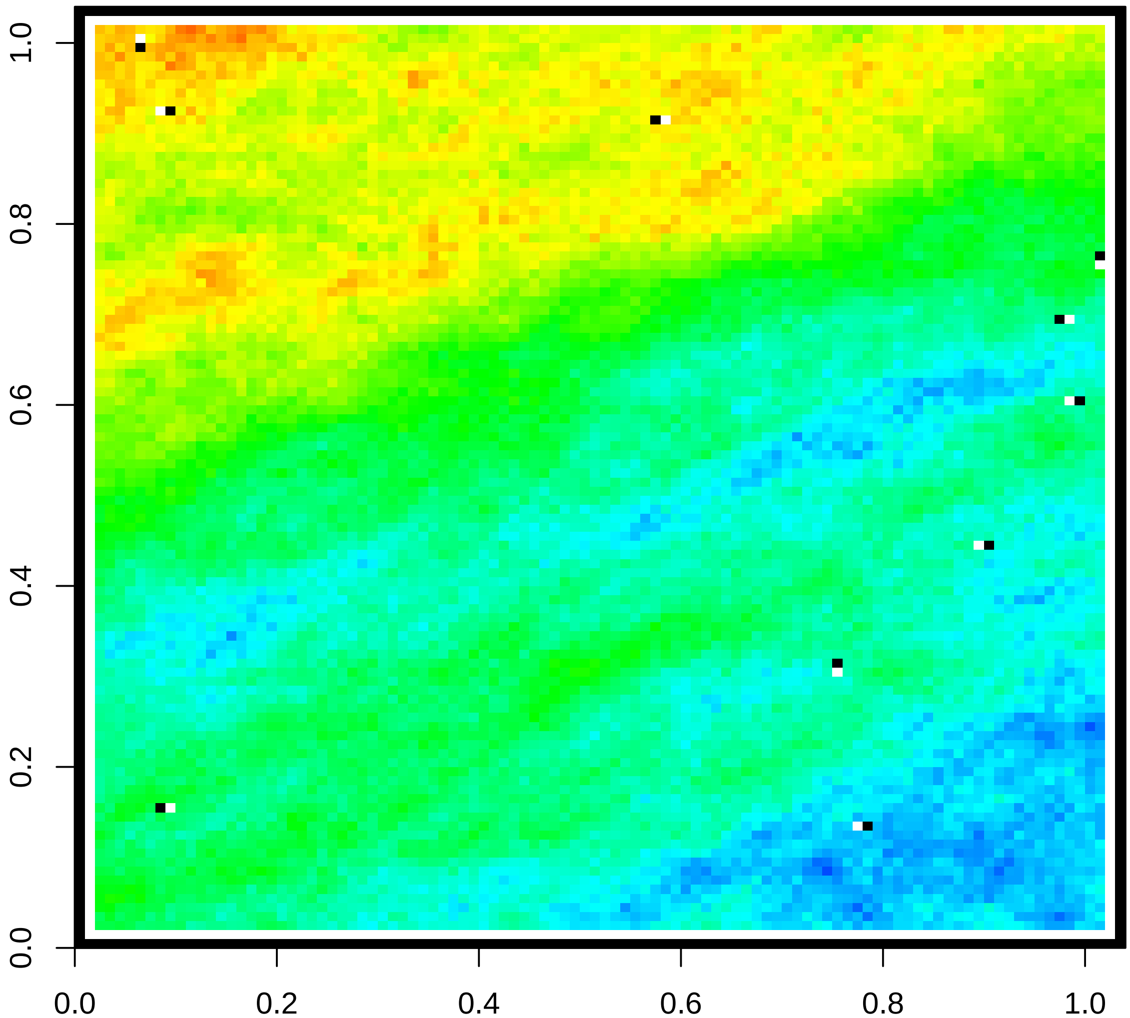
$$SE_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$SE_{\bar{Y}_T - \bar{Y}_C} = \sqrt{2041.5 \left(\frac{1}{10} + \frac{1}{10} \right)} = 20.21$$

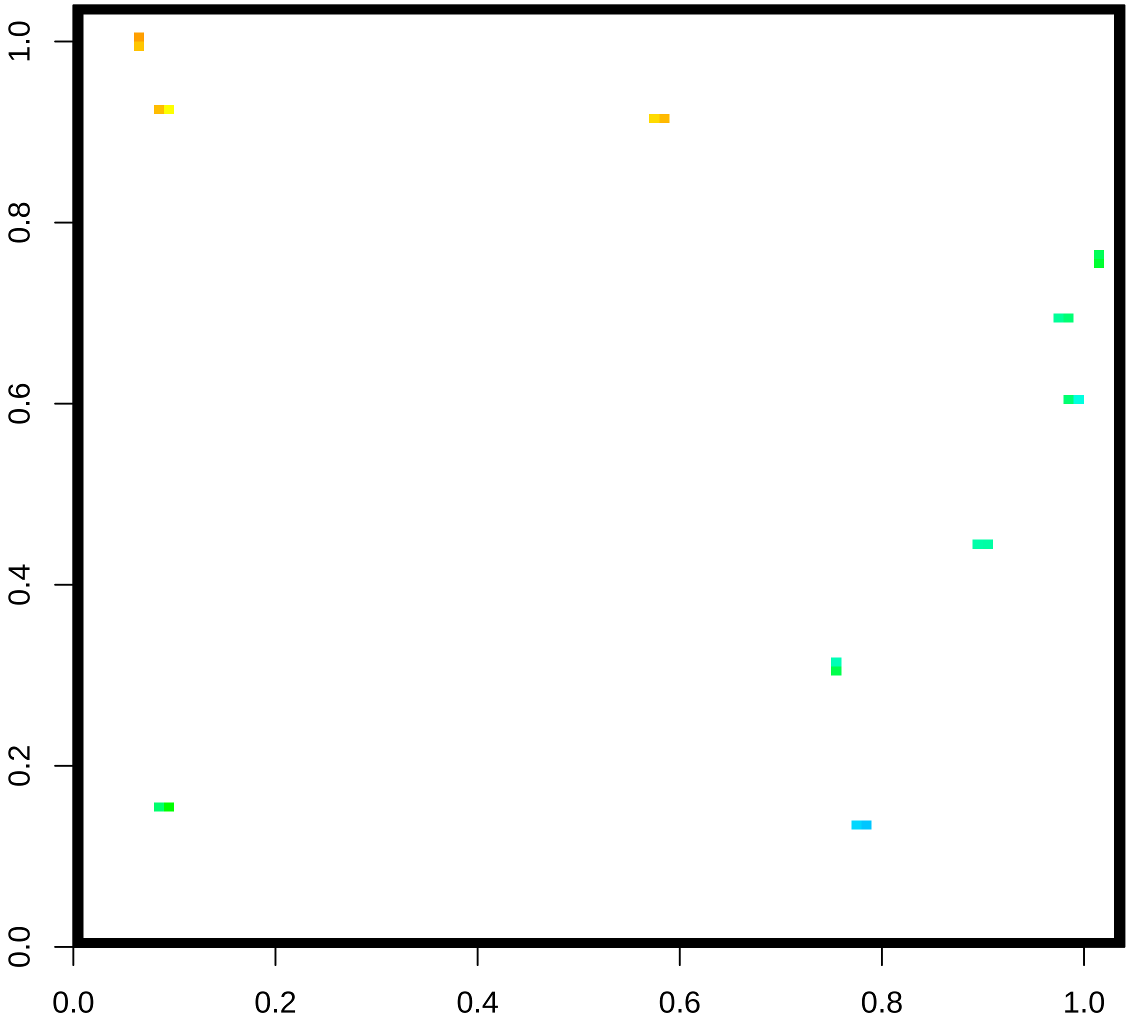
$$t = \frac{\bar{Y}_T - \bar{Y}_C}{SE_{\bar{Y}_1 - \bar{Y}_2}} = \frac{11.98}{20.21} = 0.593$$

$$df = 18$$

In a paired experimental design, we look at the difference between treatment and control from data that is naturally paired:



Only the sampled locations shown:



Unpaired			Paired	
Treatment	Control		Treatment	Control
173.9	202.0		39.6	48.5
187.3	119.9		45.5	53.4
168.0	74.1		140.9	166.0
77.1	167.1		149.5	175.3
151.2	64.7		192.2	196.5
101.5	174.8		162.6	162.0
147.9	79.0		46.6	62.2
146.0	183.6		122.0	148.2
140.8	67.5		134.7	143.3
124.3	165.5		149.7	158.7

Unpaired:
$t = 0.5929$, $df = 18$, P-value $= 0.5606$
95 % confidence interval of the difference in means:
(-30.5, 54.4)

Paired		
Treatment	Control	Difference
39.6	48.5	-8.9
45.5	53.4	-7.9
140.9	166.0	-25.1
149.5	175.3	-25.8
192.2	196.5	-4.3
162.6	162.0	0.6
46.6	62.2	-15.6
122.0	148.2	-26.2
134.7	143.3	-8.6
149.7	158.7	-9.0

$$\bar{d} = -13.08$$

$$s_d = 9.60$$

$$n = 10$$

$$SE_{\bar{d}} = 3.04$$

$\bar{d} = -13.08 \quad s_d = 9.60 \quad n = 10$

$$SE_{\bar{d}} = \frac{9.6}{\sqrt{10}} = 3.04$$

$$t = \frac{\bar{d} - \mu_0}{SE_{\bar{d}}} = \frac{-13.08}{3.04} = -4.31$$

$$df = 9$$

$$P\text{-value} = 0.00197$$

We reject the null hypothesis that our treatment has no effect on the biomass of bark beetles. We compared 10 geographically-paired plots. Treated areas had an average of 13.08 kg less bark beetles (standard error of the difference = 3.04). This difference is too large to be explained by sampling error ($P < 0.002$).

95 % confidence interval of the difference in means:

$$\bar{d} - \left(t_{0.05(2),9}\right) SE_{\bar{d}} < \mu_t - \mu_c < \bar{d} + \left(t_{0.05(2),9}\right) SE_{\bar{d}}$$

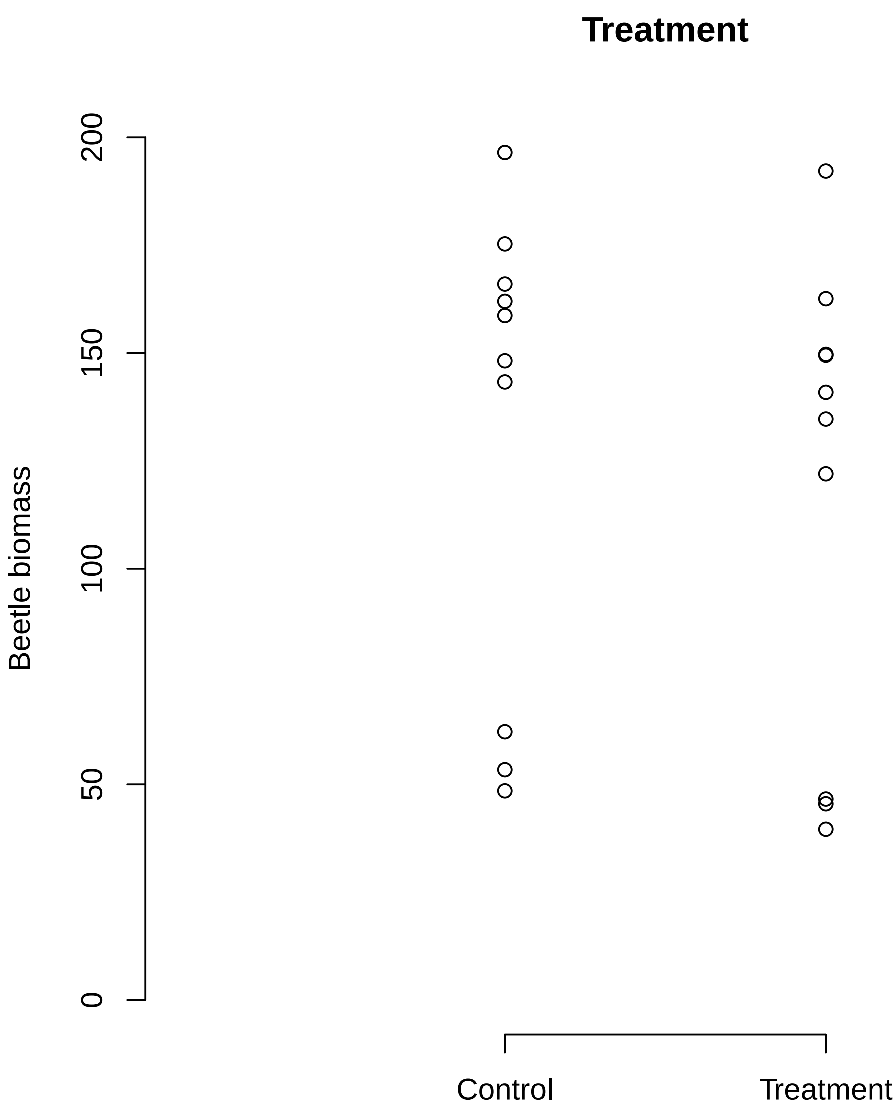
$$-13.08 - (2.26)3.04 < \mu_t - \mu_c < -13.08 + (2.26)3.04$$

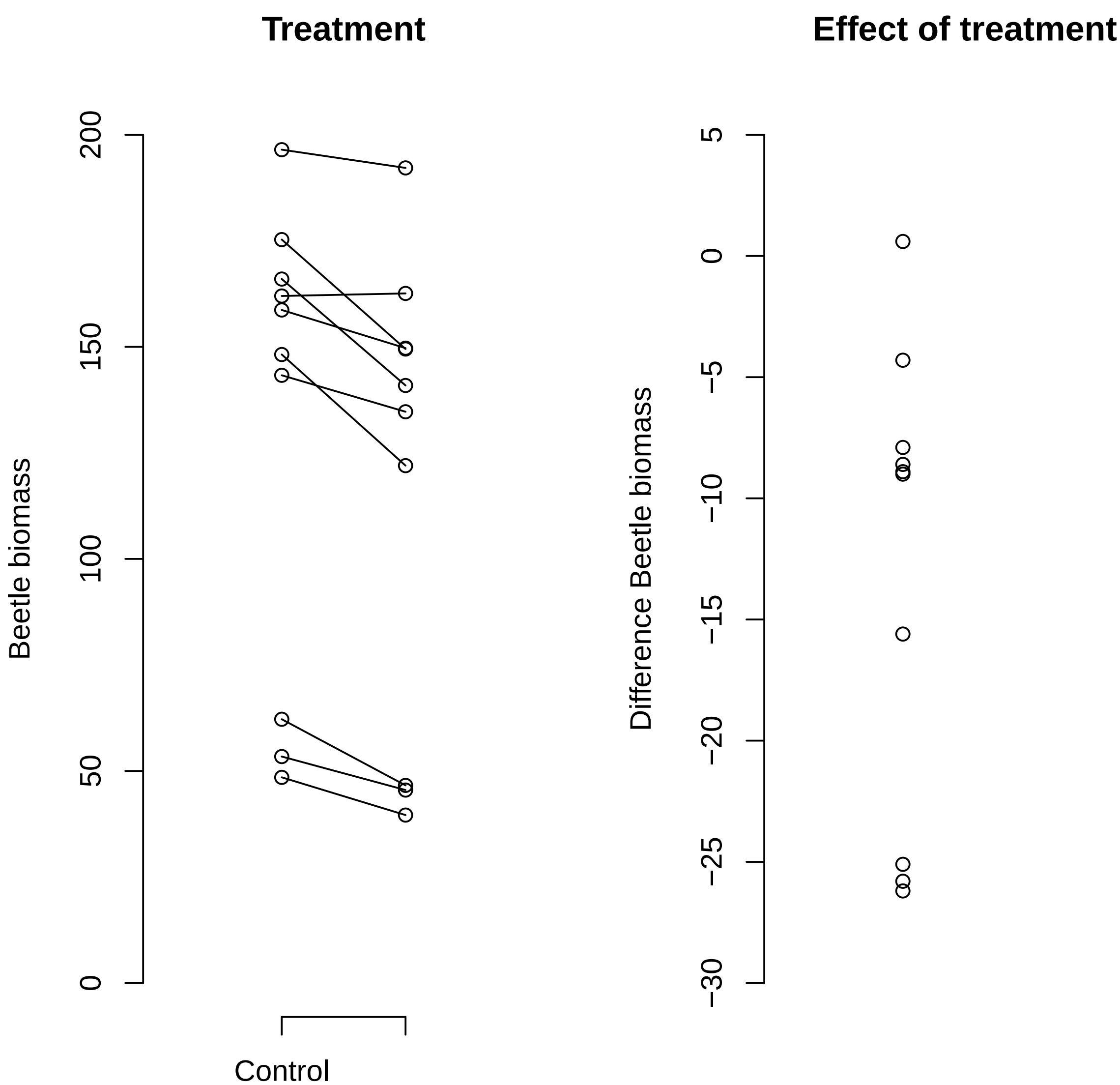
$$-19.95 < \mu_t - \mu_c < -6.21$$

Unpaired			Paired	
Treatment	Control		Treatment	Control
173.9	202.0		39.6	48.5
187.3	119.9		45.5	53.4
168.0	74.1		140.9	166.0
77.1	167.1		149.5	175.3
151.2	64.7		192.2	196.5
101.5	174.8		162.6	162.0
147.9	79.0		46.6	62.2
146.0	183.6		122.0	148.2
140.8	67.5		134.7	143.3
124.3	165.5		149.7	158.7

Unpaired:
t = 0.5929, df = 18, P-value = 0.5606
95 % confidence interval of the difference in means:
(-30.5, 54.4)

Paired: t = -4.3071, df = 9, P-value = 0.00197
95 % confidence interval of the difference in means:
(-19.95, -6.21)


Treatment
Beetle biomass
0
50
100
150
200
Control
Treatment



Treatment
Effect of treatment
Beetle biomass
200
150
100
50
0
Control
Difference Beetle biomass
5
0
−5
−10
−15
−20
−25
−30

Ignoring pairing:

Treatment	Control	Difference
39.6	48.5	-8.9
45.5	53.4	-7.9
140.9	166.0	-25.1
149.5	175.3	-25.8
192.2	196.5	-4.3
162.6	162.0	0.6
46.6	62.2	-15.6
122.0	148.2	-26.2
134.7	143.3	-8.6
149.7	158.7	-9.0

$\bar{Y}_T = 118.33 \qquad \sigma_T^2 = 2978.991 \qquad n_T = 10$

$\bar{Y}_C = 131.41 \qquad \sigma_C^2 = 3023.659 \qquad n_C = 10$

Ignoring pairing:

$\bar{Y}_T = 118.33 \qquad \sigma_T^2 = 2978.991 \qquad n_T = 10$

$\bar{Y}_C = 131.41 \qquad \sigma_C^2 = 3023.659 \qquad n_C = 10$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$s_p^2 = \frac{(9)2978.991 + (9)3023.659}{18} = 3001.325$$

$$SE_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$SE_{\bar{Y}_T - \bar{Y}_C} = \sqrt{3001.325 \left(\frac{1}{10} + \frac{1}{10} \right)} = 24.50$$

$$t = \frac{\bar{Y}_T - \bar{Y}_C}{SE_{\bar{Y}_1 - \bar{Y}_2}} = \frac{-13.08}{24.50} = -0.53$$

$$df = 18 \qquad P \approx 0.3$$

Consider paired and unpaired analyses of the same (paired) data:

Paired		**Unpaired**
Reject H_0 $t = \frac{\bar{d}}{SE_{\bar{d}}}$ $df = 9$		Do not reject H_0 $t = \frac{\bar{Y}_T - \bar{Y}_C}{SE_{\bar{Y}_1 - \bar{Y}_2}}$ $df = 18$
$t = \frac{\bar{d}}{\left(\frac{s_d}{\sqrt{n}}\right)}$ $t = \frac{-13.08}{3.04}$		$t = \frac{\bar{Y}_T - \bar{Y}_C}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ $t = \frac{-13.08}{24.50}$

If you are a doctor who wants to reduce the level of cholesterol in the bloodstream of a patient, should you prescribe exercise or drug XYZ?

(fake) data from a (fake) study:

- randomly select trial subjects,
- measure their cholesterol level,
- randomly assign them to a treatment (exercise or XYZ),
- treat for 6 months
- measure their cholesterol level after the treatment,
- **report the difference** in cholesterol level for each patient.

H_0: $\mu_e = 0$
H_A: $\mu_e \neq 0$

H_0: $\mu_d = 0$
H_A: $\mu_d \neq 0$

H_0: $\mu_e = \mu_d$
H_A: $\mu_e \neq \mu_d$

Change in cholesterol level by treatment (measured in mg/dL):

Exercise	Drug XYZ
-2.3	-5.2
-0.4	-2.6
5.0	-1.2
-13.8	0.9
-12.3	-7.2
	-4.1
	-9.2
	1.5
	-4.1
	-3.7

Exercise:

$n_e = 5$ $\bar{Y}_e = -4.76$ $s_e = 8.045$

Drug XYZ:

$n_d = 10$ $\bar{Y}_d = -3.49$ $s_d = 3.34$

Is each treatment effective?

H_0: exercise does not affect cholesterol level
$\mu_e = 0.0$
H_A: $\mu_e \neq 0.0$

H_0: drug XYZ does not affect cholesterol level
$\mu_d = 0.0$
H_A: $\mu_d \neq 0.0$

Exercise:
$n_e = 5$ $\quad \bar{Y}_e = -4.76$ $\quad s_e = 8.045$

Drug XYZ:
$n_d = 10$ $\quad \bar{Y}_d = -3.49$ $\quad s_d = 3.34$

$$t = \frac{\bar{Y} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)} =$$

$$t_e = \frac{-4.76}{\left(\frac{8.045}{\sqrt{5}}\right)} = -1.323$$

$$t_{0.05(2),4} = 2.78$$

$$t_d = \frac{-3.49}{\left(\frac{3.34}{\sqrt{10}}\right)} = -3.3061$$

$$t_{0.05(2),9} = 2.26$$
$$t_{0.01(2),9} = 3.25$$

We do not reject the null hypothesis that exercise does not have a mean effect on cholesterol ($P > 0.2$) based on these data ($n = 5$, mean change = -4.65mg/dL).

We found evidence that drug XYZ lowers cholesterol ($n = 10$, mean change = -3.49mg/dL). This drop is greater than would be expected if it were the result sampling error ($P < 0.01$).

We found evidence that drug XYZ is more effective than exercise at lowering cholesterol. The drug had a significant effect on cholesterol ($P < 0.01$), while exercise had no contribution that could not be explained by sampling error($P > 0.2$).

Exercise:

$n_e = 5 \qquad \bar{Y}_e = -4.76 \qquad s_e = 8.045$

Drug XYZ:

$n_d = 10 \qquad \bar{Y}_d = -3.49 \qquad s_d = 3.34$

$$t = \frac{\bar{Y}_e - \bar{Y}_d}{SE_{\bar{Y}_e - \bar{Y}_d}}$$

$$SE_{\bar{Y}_e - \bar{Y}_d} = \sqrt{s_p^2 \left(\frac{1}{n_e} + \frac{1}{n_d} \right)}$$

$$s_p^2 = \frac{(n_e - 1)s_e^2 + (n_d - 1)s_d^2}{n_e + n_d - 2}$$

$$s_p^2 = \frac{4(64.72) + 9(11.14)}{5 + 10 - 2} = 27.62$$

$$SE_{\bar{Y}_e - \bar{Y}_d} = \sqrt{s_p^2 \left(\frac{1}{n_e} + \frac{1}{n_d} \right)}$$

$$SE_{\bar{Y}_e - \bar{Y}_d} = \sqrt{27.62 \left(\frac{1}{5} + \frac{1}{10} \right)} = 2.88$$

$$t = \frac{-4.76 + 3.49}{2.88} = \frac{-1.27}{2.88} = 0.44$$

$$t_{0.05(2),13} = 2.16$$

Avoid indirect comparisons:
If the mean effect of A is significantly different from 0, but the mean effect of B is not significantly different from 0, we **cannot** conclude that the effect of A is larger than the effect of B.

We have to A vs B using a two sample t-test.

When can we look at two confidence intervals and figure out what hypothesis test conclusion we would have reached (if we had done t-test for a difference in the means)?